2-D Predictive Filters for Polynomial Signals With Applications to Wind Profiler Data

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Abstract

Polynomial predictors are known for their ability, in the absence of noise, to exactly predict a future value of a polynomial signal of a fixed order. One-dimensional filtering is a mature field and sophisticated filter design methods have already been heavily studied. Real world 2-D and higher order datasets are widely available for a multitude of applications. Thus, it is interesting to extend the existing one-dimensional polynomial predictors, e.g. Heinonen-Neuvo filter, to higher dimensional spaces. In this paper, we propose a novel 2-D polynomial predictor and evaluate its performance on a newly generated wind speed dataset.

1 Problem Formulation

One-dimensional polynomial predictors are linear (FIR or IIR) filters that, in the absence of noise, exactly predict a future value of a polynomial signal of a fixed order [1, 2]. Typically, the goal in designing such filters is to efficiently reduce noise in addition to performing exact prediction in the noiseless setting. The latter property implies that the filter is unbiased [3]. The noise reduction efficiency of a polynomial predictor decreases when the order of the polynomial increases or when the forward prediction step increases. This limits the usefulness of such predictors for higher order polynomials. In this paper, we consider low order 2-D polynomial predictors as they are likely to be most applicable for practical applications and can have reasonably good noise attenuation.

The theory of one-dimensional filtering is mature and sophisticated filter design methods have already been heavily studied [1, 4, 5, 6, 7, 8, 10, 11]. On the other hand, two- and higher-dimensional polynomial predictive filtering has received little attention. In this paper, we extend Heinonen-Neuvo [2] one-dimensional polynomial predictor to the 2-D space. We design a class of 2-D polynomial predictive filters with minimal support (window size). This approach can be used when designing filters for 2-D signals in a specific noise spectrum and signal characteristics.

An interesting application for such a filter predictor is a real-world dataset consisting of a 2-D field of three-dimensional vectors obtained from an acoustic wind profiler. We use this latter to study the basic properties of this 2-D polynomial predictive filter.

2 Method

Our main goal is to find the coefficients of the 2-D polynomial predictive filter, h_{ij}, $0 \leq i \leq p-1$, $0 \leq j \leq q-1$, that minimizes the noise gain. The window size is equal to pq.

Given an input $x(m,n)$ and an output $y(m,n)$, the filter operation can be formulated as follows:

$$y(m,n) = \sum_{i=0}^{p-1} \sum_{i=0}^{q-1} h_{ij} x(m-i, n-j), \tag{1}$$

where h_{ij} is the $(i,j)^{th}$ filter coefficient . In the case of a polynomial predictive filter with a predictive step (1,1) and a degree up to D, the filter coefficients are constrained by (2) for all polynomial $P(m,n)$ of degree $\leq D$ and for all m,n.

$$P(m,n) = \sum_{i=0}^{p-1} \sum_{i=0}^{q-1} h_{ij} P(m-i, n-j). \tag{2}$$

Monomials $[1, m, n, m^2, mn, n^2, \ldots, mn^{D-1}, n^D]$ form a basis of polynomials of degree $\leq D$. Thus, it is enough that (2) is satisfied for these elements. In other words, the constraint in (2) can be rewritten as

$$(m+1)^k (n+1)^l = \sum_{i=0}^{p-1} \sum_{i=0}^{q-1} h_{ij} (m-i)^k (n-j)^l, \tag{3}$$

for all (l, k) such as $0 \leq k$, $0 \leq l$ and $k + l \leq D$.

Equation (3) gives $(D+2)(D+1)/2$ linear equations for pq coefficients h_{ij}. Thus, for pq large enough ($\geq D+1$), the system of equations has at least one solution. The following filter can be optimized in order to minimize the noise gain (NG), given in (4) similar to [2].

$$NG = \sum_{i=0}^{p-1} \sum_{i=0}^{q-1} h_{ij}^2. \tag{4}$$

3 Experiments and Results

In our preliminary experiments, we explore a newly generated real dataset consisting of a two-dimensional field of three-dimensional vectors obtained from an acoustic wind profiler at several heights and time instances, i.e., wind speed vector measured at regular time intervals and at a set of different heights.The full characteristics of the dataset are provided in Table 1.

This dataset represents a good fit for our 2-D polynomial predictive filters. Our main assumption is that the wind speed is locally polynomial and this is based on the observation that no sudden changes in ordinary conditions occur in the wind speed. The dataset contains 8208 measurements at 10 minutes intervals and 9 different heights. The properties of 2-D (time, height) prediction filters can be tested by the speed-vector data. The wind speed vector consist of three measurements $speed_x$, $speed_y$ and $speed_z$. Their ranges are [-51m/s,50m/s], [-51m/s,50m/s], and [-6m/s,6m/s] respectively. In our experiments, we predicted the three components of the speed vector individually along with the speed norm, defined by $speed_{norm} = \sqrt{speed_x^2 + speed_y^2 + speed_z^2}$. The mean square errors at different heights are reported in Table 2 for 1-D and in Table 3 for 2-D.

By comparing the results from the 1-D and 2-D polynomial filter predictors for the same window size 2×2 for the 2-D and a window of size 4 in the 1-D, we see that the 2-D predictors have lower mean square errors and thus perform relatively better than the 1-D case. Using the extra dimension boosts the performance, especially for high amplitudes. In the case of 2-D polynomial predictive filter, because of the two dimensional aspect of the window, one can not predict accurate values for the first two heights i.e. 40m and 60m.

4 Conclusion

In this initial set of experiments, we have extended the use of predictive filters from 1-D to 2-D by designing a predictive filter for the wind speed data set. This first experimentation has shown that such a filter has promising performances, as it results in lower mean square errors. 2-D predictive

Table 1: Characteristics of wind speed dataset.

attribute	Characteristics
Number of time steps	8208
Time resolution	10 min
Heights range	[40m, 200m]
Heights step size	20m

Table 2: Results of 1-D polynomial filter predictor on wind speed data.

Poly degree	Window Size	Variable	40m	60m	80m	100m	120m	140m	160m	180m	200m
2	4	$speed_x$	2.62	2.84	3.31	7.36	10.83	18.70	26.63	40.68	92.61
2	4	$speed_y$	2.67	2.86	3.83	5.19	8.99	16.14	23.00	44.88	70.96
2	4	$speed_z$	1.37	1.43	1.49	1.60	1.79	2.05	2.60	3.09	3.82
2	4	$speed_{norm}$	2.39	2.59	2.82	5.76	8.52	11.92	17.67	31.72	52.95

Table 3: Results of 2-D polynomial filter predictor on wind speed data.

Poly degree	Window Size	Variable	40m	60m	80m	100m	120m	140m	160m	180m	200m
2	[2 2]	$speed_x$	Inf	Inf	1.38	3.63	6.48	10.05	18.32	29.37	64.83
2	[2 2]	$speed_y$	Inf	Inf	2.10	3.14	6.56	10.46	16.97	31.82	60.8
2	[2 2]	$speed_z$	Inf	Inf	0.40	0.46	0.58	0.70	1.01	1.38	1.93
2	[2 2]	$speed_{norm}$	Inf	Inf	1.00	3.00	5.00	6.00	11.00	22.00	36.00

filtering can be used in hybrid structures involving standard or vector median operation. Future works will focus on developing a machine learning based solution [12] for wind-speed prediction.

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