Radiation from Electric Dipole within the Time Domain

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Abstract

Radiation from electric and magnetic dipole moments was discussed quite extensively in the past and conventionally this problem was treated through time-averaged notations. However, the development of time-modulated materials and metamaterials requires transient description of radiation from elementary dipoles which will be valid not only in time-harmonic case. Here, we discuss and derive radiated power from electric dipole using elecromotive force method and prove its validity in the simulations. During the talk, we are going to present analogous results for a magnetic dipole.

1 Introduction

Radiation from an antenna is governed by the currents flowing on its surface. This surface might have a very complex shape and calculation of current distribution represents a separate and heavy problem. In order to approach such complex problem, we are going to study the simplest cases - dipole antenna, which is a very wellknown object. According to the antenna theory terminology, the radiation of the dipole is described through the concept of radiation resistance $R_{\text{rad}} = 80\pi^2(l/\lambda)^2$, where l is the effective dipole length and λ denotes the wavelength [1]. This result stems from the time-harmonic calculation of the time-averaged radiated power and the time average of the square of the instantaneous electric current carried by the dipole. In other words,

$$R_{\text{rad}} = \langle P_{\text{rad}} \rangle / \left\langle i(t)^2 \right\rangle, \tag{1}$$

where the brackets ($\langle ... \rangle$) denote time averaging over one period. Such description of radiation is applicable only for time-harmonic currents, which is not always the case in arbitrary time-modulated structures [2, 3]. Here, we show how to derive the correct expression for radiated power from elementary dipoles that can be used for arbitrary function of current. We also verify our theoretical results with simulations.

2 Theory of Electric Dipole Radiation

Let us consider an oscillating Hertzian dipole with the current density

$$\mathbf{J} = Il\delta(\mathbf{r})\,\mathbf{a}_z. \tag{2}$$

It is assumed the the dipole of length l is directed along axis z ($\mathbf{a}_z$ is the unit vector along the z-axis) and carries time-harmonic current $i(t) = I\cos(\omega t)$. Placing this dipole in the origin and using the induced electromotive force method, we find the instantaneous radiated power from such Hertzian dipole. We know that the theta component ($\mathbf{a}_\theta$) of the electric field generated by the Hertzian dipole is expressed as

$$\mathbf{E} = j\omega\mu_0 \frac{Il}{4\pi}\left[1 + \frac{c}{j\omega r} - \frac{c^2}{\omega^2 r^2}\right]\frac{e^{\left(\frac{-j\omega r}{c}\right)}}{r}\sin\theta\,\mathbf{a}_\theta, \tag{3}$$

where ω is the angular frequency, μ_0 and ε_0 represent the permeability and permittivity of vacuum, respectively, and c denotes the speed of light. We are interested in examining the electric field at very small distances from the dipole: $r \to 0$. While the total electric field is singular at the source location, there is a non-singular component. It can be found expanding the exponential function in the above equation:

$$e^{-j\omega r/c} \approx 1 - j\frac{\omega r}{c} - \frac{\omega^2 r^2}{2c^2} + j\frac{\omega^3 r^3}{6c^3}. \tag{4}$$

Keeping these four terms in the expansion is sufficient to find the non-singular part. To do that, we substitute Eq. (4) into Eq. (3) and obtain

$$\mathbf{E}_{\rm ns} = \frac{(j\omega)^2 \mu_0}{6\pi c} Il\ \mathbf{a}_z. \tag{5}$$

At this point, we can use the inverse Fourier transform to return to the time domain. In Eq. (5), $(j\omega)^2$ corresponds to the second time derivative of the electric current. Thus,

$$\mathbf{E}_{\rm ns}(t) = \frac{\mu_0}{6\pi c}\frac{d^2 i(t)}{dt^2} l\ \mathbf{a}_z. \tag{6}$$

Knowing the time-domain expression of the non-singular component of the electric field parallel to the dipole moment, we obtain the corresponding electromotive force (assuming that the dipole size is negligibly small as compared with all relevant wavelengths):

$$\mathscr{E}(t) = -\int_l \mathbf{E}_{\rm ns}(t) \cdot \mathbf{dl} = -\frac{\mu_0}{6\pi c}\frac{d^2 i(t)}{dt^2} l^2. \tag{7}$$

Expressing the electric current in terms of the corresponding electric dipole moment,

$$\frac{d^3 p(t)}{dt^3} = l\frac{d^2 i(t)}{dt^2}, \tag{8}$$

the formula for the electromotive force reduces to

$$\mathscr{E}(t) = -\frac{\mu_0}{6\pi c}\frac{d^3 p(t)}{dt^3} l. \tag{9}$$

After some simple algebraic manipulations and recalling that $i(t) = (1/l)\ dp(t)/dt$, finally, we derive the instantaneous radiated power radiated from the Hertzian dipole:

$$P_{\rm rad}(t) = \mathscr{E}(t) i(t) = -\frac{\mu_0}{6\pi c}\frac{d\mathbf{p}(t)}{dt} \cdot \frac{d^3\mathbf{p}(t)}{dt^3}. \tag{10}$$

Note that the direct inverse Fourier transform of the usual expression for power radiated from a time-harmonic dipole source gives a different result, where the oscillating part of the power is missing.

3 Simulation Results

In order to verify the theory, we have simulated a half wavelength dipole and studied radiation from it. A dipole antenna at the resonance, after the transition perios, radiates all the supplied power, since the reactive power is 0. Figure 1 shows instanteneous power balance for a resonant dipole. The length of the dipole is chosen so that the dipole resonates at about 1.5 GHz. The radius of the cylinder is supposed to be considerably smaller than the total length. As the figure shows, the instantaneous input power $P_{\rm in}(t)$ (blue curve) and the radiated power $P_{\rm rad}(t)$ (red curve) are equal in magnitude with a 180-degree phase difference. Therefore the summation of the radiated power and the input power is zero. The orange dased curve represents radiated power given by formula,

$$P_{\rm rad}(t) = \frac{\mu_0}{6\pi c}\ddot{\mathbf{p}}(t) \cdot \ddot{\mathbf{p}}(t), \tag{11}$$

which is considered as conventional instanteneous radiated power [4, 5].

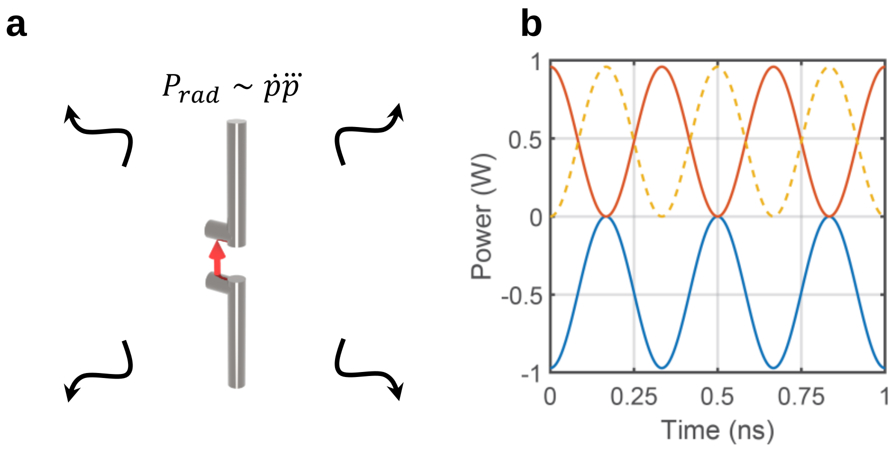

Fig. 1: Instantaneous power balance for transmitting regime regarding a resonant dipole antenna. (a) Electric dipole is fed in the center by a time-harmonic source representing transmitting regime. (b) Resonant dipole: the length of the cylindrical wire, made of perfect conductor, is $l = 93.9$ mm and the radius of the wire is $r = 0.3$ mm. The red solid curve shows the instantaneous radiated power, while the blue dot-dashed curve corresponds to a supplied power. Additionally, the dashed orange curve represents the conventional interpretation of the instantaneous radiated power, i.e., $P(t) = (\mu_0/6\pi c)\ddot{\mathbf{p}}(t)\cdot\ddot{\mathbf{p}}(t)$, which is seen to be incompatible with the instantaneous conservation of energy.

4 Conclusions

We have derived the general time-domain equation for radiating dipole moments and, in particular, considered a time-modulated cylindrical wire antenna as an example. The validity of the proposed model was confirmed in transmitting regime where the wire antenna is excited by an external lumped source. In the presentation we are going to show analogous derivation for magnetic dipole and discuss its application to time-modulated meta-atoms.

References

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