Theory of Characteristic Modes for Conducting and Dielectric Objects

Henrik Wallén Pasi Ylä-Oijala

Department of Electronics and Nanoengineering
Aalto University, Espoo, Finland
henrik.wallen@aalto.fi

Abstract

In this presentation, we briefly explain the theory of characteristic modes for conducting objects and discuss some of the difficulties and recent progress in formulating the theory for dielectric objects.

1 Introduction

The theory of characteristic modes (TCM) is currently an active research topic in antenna design [1]. For an arbitrary geometry and a fixed frequency, TCM provides a set of characteristic eigencurrents and corresponding eigenvalues that provide some excitation independent insight into the antenna design process. Before designing and impedance matching a feeding structure, the characteristic modes can be used to evaluate how well the antenna could radiate. Based on the characteristic eigencurrents it is also easier to determine where the feed(s) should be placed to excite certain modes and perhaps suppress other ones.

For perfectly electrically conducting (PEC) objects, the theory is well established and characteristic mode analysis is implemented in several commercial simulators. However, it has been a somewhat open problem how TCM should be formulated for non-PEC objects.

2 EFIE-based TCM for PEC objects

For perfectly electrically conducting (PEC) objects, TCM is typically formulated using the electric field integral equation (EFIE) as presented by Harrington and Mautz [2].

EFIE is a surface integral equation where the scattered electric field due to a PEC object is expressed in terms of the surface current $\mathbf{J}$ on the object. Thus, the EFIE operator is an impedance operator $\mathcal{Z}(\mathbf{J})$ mapping a surface current into an electric field on the same surface. This operator is complex and symmetric, and so its real and imaginary parts, $\mathcal{Z} = \mathcal{R} - i\,\mathcal{X}$, are real and symmetric. Moreover, the real part $\mathcal{R}$ is positive definite and related to radiated power.

The characteristic currents $\mathbf{J}_n$ and their corresponding eigenvalues λ_n are solutions of the generalized eigenvalue equation [2]

$$\mathcal{X}(\mathbf{J}_n) = \lambda_n \mathcal{R}(\mathbf{J}_n). \tag{1}$$

From this formulation follows the following important properties of the characteristic modes:

1. The eigenvalues λ_n and eigencurrents $\mathbf{J}_n$ are real.

2. The radiated far fields of the eigencurrents $\mathbf{J}_n$ are orthogonal and any radiated or scattered field can be expressed as a linear combination of these characteristic modes.

3. Each eigenvalue give the ratio of reactive and radiated power of the corresponding eigencurrent, $\lambda_n = P_n^{\text{reac}}/P_n^{\text{rad}}$, thus measuring how well each mode radiates.

3 TCM formulations for dielectric objects

The so called PMCHWT formulation is perhaps the most popular surface integral equation for scattering by dielectric objects. Based on a symmetrized version of the PMCHWT operator, Chang and Harrington [3] presented a TCM formulation that mimics the formulation for PEC [2]. Since the operator is symmetric and its real part appears to be positive definite, it gives real characteristic eigenvalues and eigencurrents. However, it has recently been demonstrated that the Chang–Harrington formulation suffers from spurious modes [4].

The problem with the Chang–Harrington formulation is that the weight operator, corresponding to operator $\mathcal{R}$ in (1), contains a mixture of surface integral operators for both the interior and the exterior of the dielectric object. Hence, it turns out that the formulation produces both the wanted characteristic modes and also a set of modes that neither agree with the volume integral equation formulation in [5] nor with the analytical solution for dielectric spheres [6].

One way to avoid the spurious modes is to use the interior operators only to eliminate the equivalent magnetic surface current [7], but the crucial step in the formulation is actually to select the weight operator so that it is properly related to radiated power [8].

References

[1] Y. Chen and C.-F. Wang, *Characteristic Modes: Theory and Applications in Antenna Engineering*. Wiley, 2015.

[2] R. F. Harrington and J. R. Mautz, "Theory of characteristic modes for conducting bodies," *IEEE Trans. Antennas Propag.*, vol. 19, no. 5, pp. 622–628, Sep. 1971.

[3] Y. Chang and R. F. Harrington, "A surface formulation for characteristic modes of material bodies," *IEEE Trans. Antennas Propag.*, vol. 25, no. 6, pp. 789–795, Nov. 1977.

[4] Z. T. Miers and B. K. Lau, "Computational analysis and verifications of characteristic modes in real materials," *IEEE Trans. Antennas Propag.*, vol. 64, no. 7, pp. 2595–2607, Jul. 2016.

[5] R. F. Harrington, J. R Mautz, and Y. Chang, "Characteristic modes for dielectric and magnetic bodies," *IEEE Trans. Antennas Propag.*, vol. 20, no. 2, pp. 194–198, Mar. 1972.

[6] H. Wallén, P. Ylä-Oijala, D. C. Tzarouchis, and A. Sihvola, "Mie scattering and characteristic modes of lossy dielectric objects," in *2nd URSI Atlantic Radio Science Meeting (AT-RASC 2018)*, Gran Canaria, Spain, Sep. 2018.

[7] P. Ylä-Oijala, H. Wallén, D. C. Tzarouchis, and A. Sihvola, "Surface integral equation-based characteristic mode formulation for penetrable bodies," *IEEE Trans. Antennas Propag.*, vol. 66, no. 7, pp. 3532–3539, Jul. 2018.

[8] P. Ylä-Oijala, "Generalized Theory of Characteristic Modes," *IEEE Trans. Antennas Propag.*, vol. 67, no. 6, pp. 3915–3923, Jun. 2019.